

(1) Find  $(AB)^{-1}$  where  $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$

(2) Compute the sum:  $\sum_{r=1}^{20} \frac{6}{(3r-1)(3r+2)}$

(3) Solve the equation  $z^3 = 2 + 2i$

(4) Test the series (a)  $\sum_{n=1}^{\infty} \frac{3^n}{(n+2)!}$

(b)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3 + 2}$

(5) If  $f(x, y) = e^{xy}$ . Find  $f_x$ ,  $f_{xx}$ ,  $f_y$  and the maximum (minimum) values

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(1) Find  $(A-B)^{-1}$  where  $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$

(2) Compute the sum of the series  $\sum_{r=1}^{20} \frac{8}{(4r-3)(4r+1)}$

(3) Find  $u(x,y)$  and  $v(x,y)$  of the function  $f(z) = e^{2z}$  and show that  $u_{xx} + u_{yy} = 0$

(4) Test the series (a)  $\sum_{n=1}^{\infty} n \left(\frac{2}{3}\right)^n$  (b)  $\sum_{n=1}^{\infty} \frac{(-3)^n}{(2n+1)^n}$

(5) Find the extrema of the function  $f(x,y) = x^2 + 3y^2 + 2x - 12y + 2$

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(1) Find  $A^t \cdot B$ ,  $B^t \cdot A$  where  $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 & 1 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}$

(2) Compute the sum:  $\sum_{r=1}^{20} (r+1)(2r+3)$

(3) Solve the equation  $z^4 = -16i$

(4)(a) Test  $\sum_{n=1}^{\infty} \frac{2^n}{2n+1}$  (b) Determine the interval of convergence of  $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+3} (x-2)^n$

(5) If  $f(x,y) = 3y + y^3 - 3yx^2$ . Show that  $f_{xx} + f_{yy} = 0$

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الزمن: 3 ساعات

الامتحان مكون من (5) أسئلة في صفحة واحدة و المطلوب الإجابة عن كل الأسئلة

(1)(a)Find AB and  $|AB|$  where  $A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \\ 3 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 0 & 3 \end{bmatrix}$

(b)Test the series: (i)  $\sum_{n=1}^{\infty} \frac{n}{n^4 + 4}$       (ii)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3 + 4}$

(2)(a)Determine the interval of convergence of the series  $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n+1}} (x-2)^n$

(b)Compute the sum of the series  $\sum_{r=1}^{20} (r+2)(2r+3)$

(c)Find  $u(x,y)$ ,  $v(x,y)$  of the complex function  $f(z) = z + \sin z$  and  $u_{xx} + u_{yy}$

(3)(a)Obtain the maximum and minimum values of the function:

$$f(x,y) = x^3 + y^2 - 27x - 4y + 3$$

(b)Find the envelope of the curves:  $(x-\alpha)^2 + y^2 = 4\alpha$

(c)Verify Euler,s theorem for  $f(x,y,z) = x^2 + y^2 - 3z^2 - 4xy$ . Also, find  $\nabla f$

(4)(a)Find  $\bar{U} \cdot \bar{V}$ ,  $\bar{U}_x \bar{V}$  where  $\bar{U} = 2i + j + 2k$  and  $\bar{V} = i + 2j - 2k$ . Also, find the angle between  $\bar{U}, \bar{V}$ .

(b)If  $\bar{U} = (xyz)i + (2xz + y)j + (z \sin y)k$ . Find  $\nabla \cdot \bar{U}$  and  $\nabla \times \bar{U}$

(c)Evaluate  $\int_{(0,0)}^{(1,1)} (3x^2 + 4y) dx + (4x + 2y) dy$ , through the curve  $y = x^3$

(5)(a)Evaluate the integrals: (i)  $\int_0^2 \int_0^x (3x + 3y^2) dy dx$       (ii)  $\int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{x^2 + y^2} dy dx$

(b)Verify Green's theorem for the integral  $\oint_C (3x^2 + 2y) dx + (2x^2 + y) dy$ ,

where C is formed by:  $y = \sqrt{4 - x^2}$ ,  $y = 0$